Color-flavor locked strangelets and their detection

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Abstract. Strange quark matter in a color-flavor locked state is significantly more bound than "ordinary" strange quark matter. This increases the likelihood of strangelet metastability or even absolute stability. Properties of color-flavor locked strangelets are discussed and compared to ordinary strangelets. Apart from differences in binding energy, the main difference is related to the charge. A statistical sample of strangelets may allow experimental distinction of the two. Preliminary estimates indicate that the flux of strangelets in galactic cosmic rays could be sufficient to allow for strangelet discovery and study in the upcoming Alpha Magnetic Spectrometer AMS-02 cosmic ray experiment on the International Space Station.

1. Introduction

It has long been known that phenomenological strong interaction models, most notably the MIT bag model, allow absolute stability (energy per baryon below 930 MeV) of three flavor quark matter for certain ranges of parameters, and metastability for a wider parameter span. While bag model calculations are clearly only a crude approximation to full (but untractable) QCD, the confirmation of strange matter (meta)stability would have important consequences. For instance strange matter stability would imply that "neutron stars" are actually quark stars (strange stars), and metastability could also significantly change the physics of neutron star interiors.

Even if quark matter is (meta)stable in bulk, finite size effects (surface and curvature energies) increase the energy per baryon for small lumps of three flavor quark matter, called strangelets. This makes it less likely to form such objects in heavy-ion collisions, and such formation is also hindered by the high entropy/temperature environment, which destabilizes strangelets further ("making ice-cubes in a furnace"). Nevertheless, tiny amounts of strangelets might be formed in colliders, and the detection of one would be the ultimate smoking gun for the quark-gluon plasma [1].

The probability of strangelet (meta)stability is increased with the recent demonstration that quark matter at high density may be in a so-called color-flavor locked phase where quarks with different color and flavor quantum numbers form Cooper pairs with pairing energy Δ perhaps as high as 100 MeV [2]. Such a state is significantly more bound than ordinary quark matter, and this increases the likelihood that quark matter composed of up, down, and strange quarks may be metastable or even absolutely stable. In other words color-flavor locked quark matter rather than nuclear matter could be the ground state of hadronic matter.

In the following I shall briefly summarize the physics of "ordinary" strangelets, followed by a summary of recent work on color-flavor locked strangelets. Finally, I discuss the potential of experimental discovery of strangelets with the AMS-02 detector on the International Space Station, which may even (by studying the mass-charge relation) allow a distinction between ordinary and color-flavor locked strangelets.

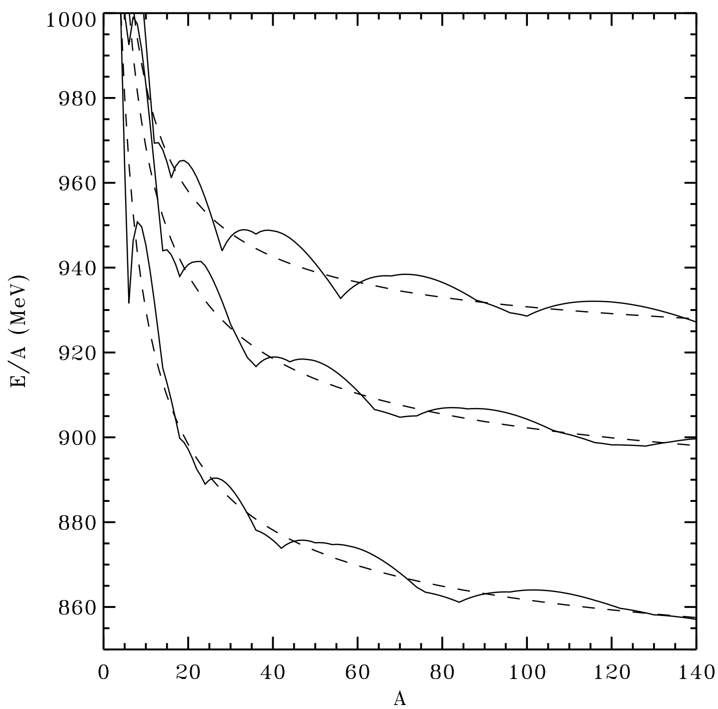

Figure 1. Energy per baryon in MeV is plotted as a function of baryon number for "ordinary" strangelets with s-quark mass of 50, 150, and 300 MeV (bottom to top). The bag constant is $B = (145\ \mathrm{MeV})^4$. Mode filling within the MIT bag model corresponds to the solid curves, whereas dashed curves were calculated from the smoothed density of states within the multiple reflection expansion. It is clearly seen how E/A grows from a bulk value (c.f. Fig. 2) at high A to a value significantly increased by finite size effects at low A. It is also seen how the multiple reflection expansion including volume, surface, and curvature terms (dashed curves) gives an excellent reproduction of the overall behavior of E/A. The upper set of curves (for $m_s = 300$ MeV) essentially corresponds to two flavor quark matter, since only up- and down-quarks are energetically favorable for such a high strange quark mass. The bag constant chosen is close to the lower bound permitted if nuclei should remain stable against direct decay into two flavor quark matter (decay into strangelets is forbidden since it requires a high order weak interaction to create strange quarks).

2. Ordinary strangelets

Strangelet properties are usually studied by filling up, down, and strange quark energy levels in a spherical bag with MIT bag model boundary conditions [3]. Results of such calculations are shown in Figure 1. Shell structure reminiscent of nuclear or atomic physics is evident, but the overall picture is an energy per baryon that decreases from small to large baryon number A, saturating at a bulk value for large A.

The general behavior can be understood within a multiple reflection expansion framework. Here the energy of a system composed of quark flavors i is given by

$$E = \sum_i (\Omega_i + N_i \mu_i) + BV, \tag{1}$$

where Ω_i, N_i and μ_i denote thermodynamic potentials, total number of quarks, and chemical potentials, respectively. B is the bag constant, V is the bag volume. The thermodynamical quantities can be derived from a density of states of the form [4] $\frac{dN_i}{dk} = 6\left\{\frac{k^2 V}{2\pi^2} + f_S\left(\frac{m_i}{k}\right) kS + f_C\left(\frac{m_i}{k}\right) C + ...\right\}$, where a sphere has area $S = 4\pi R^2$ and curvature $C = 8\pi R$. For the MIT-bag model $f_S(m/k) = -\left[1 - (2/\pi)\tan^{-1}(k/m)\right]/8\pi$ [5] and $f_C(m/k) = \left[1 - 3k/(2m)\left(\pi/2 - \tan^{-1}(k/m)\right)\right]/12\pi^2$ [6]. The number of quarks of flavor i is $N_i = \int_0^{p_{Fi}} (dN_i/dk)dk = n_{i,V}V + n_{i,S}S + n_{i,C}C$, and the thermodynamic potentials are $\Omega_i = \int_0^{p_{Fi}} (dN_i/dk)(\epsilon_i(k) - \mu_i)dk = \Omega_{i,V}V + \Omega_{i,S}S + \Omega_{i,C}C$, where $\epsilon_i(k) = (k^2 + m_i^2)^{1/2}$. The expressions obey $\partial\Omega_i/\partial\mu_i = -N_i$, and $\partial\Omega_{i,j}/\partial\mu_i = -n_{i,j}$. The ground state strangelet with a given A is found by minimizing E with respect to R and N_i for fixed A. As seen from the full lines in Figure 1 such an expansion completely reproduces the average behavior of E/A versus A.

3. Color-flavor locked strangelets

Strangelets in a color-flavor locked state can be described in a framework similar to the one summarized above for ordinary strangelets. This was first attempted in Ref. [7] (a different approach to finite size effects in the case of two flavor color superconducting quark matter was recently presented in [8]). In [7] the total energy (mass) of a strangelet was written in a manner similar to Eq. (1) as

$$E = \sum_i (\Omega_i + N_i \mu_i) + (\Omega_{\text{pair},V} + B)V, \tag{2}$$

where $\Omega_{\text{pair},V} \approx -3\Delta^2\mu^2/\pi^2$ is the binding energy from pairing (μ is the average quark chemical potential), and the thermodynamic potential of quark flavor i is a sum of volume, surface, and curvature terms as discussed above.

An important difference relative to non-CFL calculations is that all quark Fermi momenta in CFL strange quark matter are equal. This property leads to charge neutrality in bulk without any need for electrons [9], and it is due to the fact that pairing happens between quarks of different color and flavor, and opposite momenta $\vec{p}$ and $-\vec{p}$. It is therefore energetically favorable to fill all Fermi seas to the same Fermi

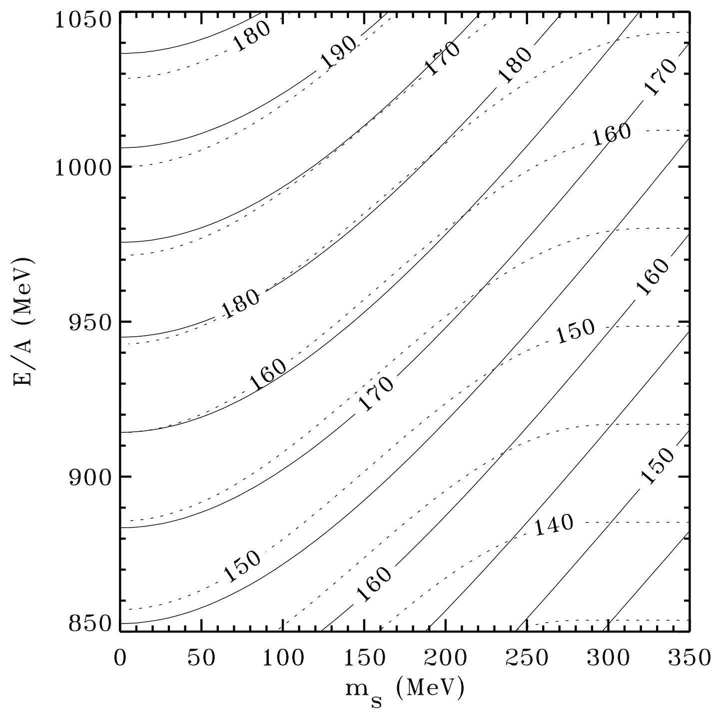

Figure 2. Bulk energy per baryon for strange quark matter as a function of strange quark mass, for several choices of $B^{1/4}$ in MeV. Dotted curves correspond to "ordinary" strange quark matter. Solid curves include the effect of color-flavor locking with a gap parameter $\Delta = 100$ MeV. It is seen that a typical energy gain for fixed B is of order 100 MeV per baryon. Notice how E/A for "ordinary" strange quark matter saturates at high m_s for fixed B, whereas the corresponding curve for CFL-strange quark matter continues to rise. This difference is due to the fact that increasing m_s makes ordinary quark matter shift from three to two flavors, whereas the CFL-phase keeps equal numbers of up, down and strange quarks to maximize the pairing energy even at quite high m_s.

momentum, p_F. For bulk quark matter the energy per baryon with and without color-flavor locking are compared in Fig. 2. For $\Delta = 100$ MeV the gain in energy per baryon is of order 100 MeV for realistic values of the s-quark mass.

As illustrated in Figure 3, color-flavor locked strangelets have an energy per baryon, E/A, that behaves much like that of ordinary strangelets as a function of A. For high A a bulk value is approached, but for low A the finite-size contributions from surface tension and curvature significantly increase E/A, making the system less stable. The main difference from ordinary strangelet calculations is the overall drop in E/A due to the pairing contribution, which is of order 100 MeV per baryon for $\Delta = 100$ MeV for fixed values of m_s and B. Since $\Omega_{\mathrm{pair},V} \propto \Delta^2$, the actual energy gain is quite dependent on the choice of Δ.

The charge properties of ordinary strangelets and CFL strangelets are quite different. Both have a very small charge per mass unit relative to nuclei, but the exact

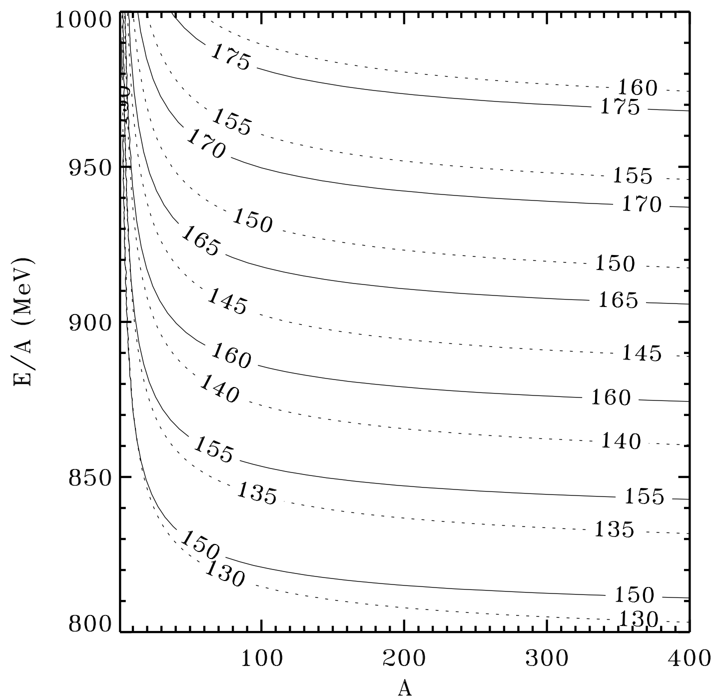

Figure 3. Energy per baryon in MeV as a function of A for ordinary strangelets (dotted curves) and CFL strangelets (solid curves) for $B^{1/4}$ in MeV as indicated, $m_s = 150$ MeV, and $\Delta = 100$ MeV. All calculations are performed within the multiple reflection expansion of the MIT bag model. The dotted curve for $B^{1/4} = 145$ MeV corresponds to the middle dashed curve in Figure 1.

relation may provide a way to test color-flavor locking experimentally if strangelets are found in accelerator experiments or (perhaps more likely) in cosmic ray detectors. With $m_{150} \equiv m_s/(150\text{ MeV})$ ordinary strangelets have (roughly) [10, 5, 11]

$$Z \approx 0.1 m_{150}^2 A, \qquad A \ll 10^3, \tag{3}$$

$$Z \approx 8 m_{150}^2 A^{1/3}, \qquad A \gg 10^3, \tag{4}$$

whereas CFL strangelets are described by [7]

$$Z \approx 0.3 m_{150} A^{2/3}. \tag{5}$$

This relation can easily be understood in terms of the charge neutrality of bulk CFL strange quark matter [9] with the added effect of the suppression of s-quarks near the surface, which is responsible for (most of) the surface tension of strangelets. This leads to a reduced number of negatively charged s-quarks in the surface layer; thus a total positive quark charge proportional to the surface area or $A^{2/3}$.

A similar effect becomes important in ordinary strangelets, meaning that the standard $A^{1/3}$-result, Eq. (4), breaks down at very high A [12]. This effect is large enough to rule out a potential disaster scenario, where negatively charged strangelets produced in heavy ion colliders could grow by nucleus absorption and swallow the Earth [13]. While ordinary strange quark matter can be negatively charged in bulk if the one-gluon exchange α_S is very prominent [10], the added positive surface charge due to massive s-quark suppression is sufficient to make the overall quark charge positive for a large range of A, thus preventing any such disaster [12].

Only first steps have been made in the effort to describe properties of color-flavor locked strangelets, and there is room for improvement. While finite-size effects were included in the free quark energy calculations, such (unknown) higher order terms were not taken into account in the pairing energy. This approximation is only warranted as long as $\Omega_{\rm pair}$ itself is a perturbation to $\Omega_{\rm free}$. The discreteness of quark energy levels was only taken into account in an average sense via the smoothed density of states given as a sum of volume, surface and curvature terms. This is an excellent approximation to the average strangelet properties (c.f. Fig. 1) [6], but it misses the interesting stabilizing effects near closed shells [14, 15] that could make certain baryon number states longer lived than one might expect from Figure 3. Discreteness should also be considered more carefully in the treatment of pairing for small A. Most important the MIT bag model with $\alpha_S = 0$ is only a crude phenomenological approximation to strong interaction physics; it is not QCD.

4. Strangelet detection at The International Space Station

While strangelet formation in the cosmological quark-hadron phase transition seems less likely than originally believed [16, 1], a significant flux of cosmic ray strangelets is expected from another source if strange matter is stable, namely collisions of binary compact star systems containing strange stars. If strange quark matter is the ground state of hadronic matter at zero pressure, it will be energetically favorable to form strange stars rather than neutron stars, and it would be expected that all the objects normally associated with neutron stars (pulsars and low-mass x-ray binaries) would actually be strange stars [17]. Several pulsars are observed in binary systems containing another compact star. Such binaries move in elliptical orbits, spiraling closer to each other because the system loses energy by gravity wave emission. Ultimately the stars collide. The expected rate of binary collisions in our Galaxy is of order 10^{-4} year^{-1}.

Numerical studies have followed the late stages of inspiral in systems composed of two neutron stars or neutron stars orbiting black holes or white dwarfs. No detailed calculations have been done for systems containing strange stars, and since there are significant differences in the equation of state it may be dangerous to rely on existing models. Nevertheless, the release of a fraction of a solar mass seems to be a generic feature. Most collisions seem to release between 10^{-4} and $10^{-1}M_\odot$, where $M_\odot$ is the solar mass in connection with the actual collision and via tidal disruption in the late stages of inspiral. Lumps of matter released during the tidal disruption phase are expected to be macroscopic. Simple estimates balancing the tidal force with the surface tension force of strange quark matter leads to a typical fragment baryon number of order 10^{38}.

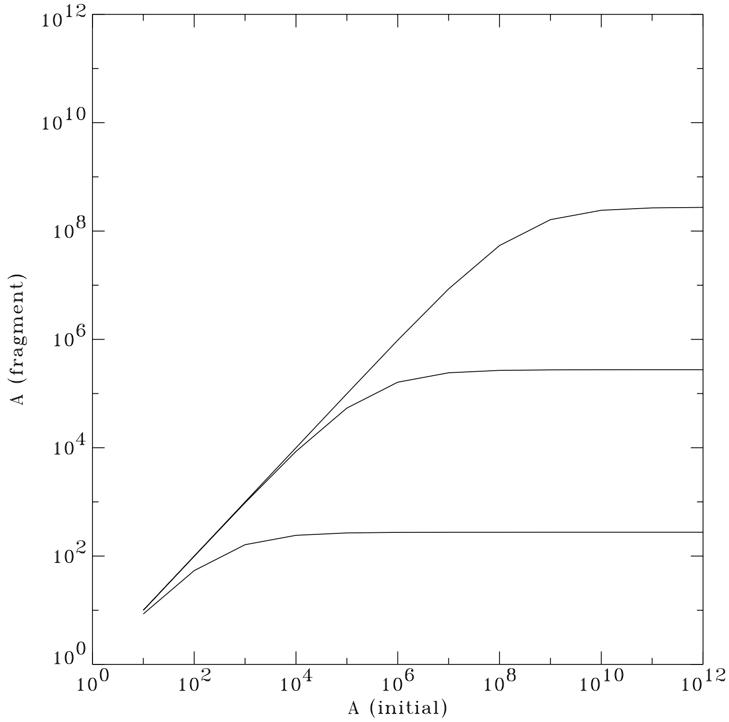

Figure 4. The typical A-value of strangelet fragments as a function of the A-value of colliding strangelets, for collision energies of 10^{-6}, 10^{-4}, and 10^{-2} of the rest mass energy (top to bottom). Calculations were performed for non-CFL strangelets with massless quarks and $B^{1/4} = 145$ MeV, but similar results are obtained for other parameter choices. Regardless of initial size, small lumps in the mass-range detectable from the International Space Station are abundantly formed for collisions with speed comparable to orbital speeds in compact binary star systems ($E_{\rm kinetic} \approx 0.01mc^2$).

A significant fraction of the tidally released material is originally trapped in orbits around the binary stars. Typical orbital speeds of the lumps are $0.1c$, and collisions among lumps are frequent. Assuming the kinetic energy in these collisions mainly goes to fragmentation of the lumps into smaller strangelets (i.e. that the kinetic energy is used to the extra surface and curvature energies necessary for forming N lumps of baryon number A/N from the original baryon number A), the resulting strangelet distribution peaks at mass numbers from a few hundred to about 10^3 as shown in Figure 4.

This is within the interesting regime for the upcoming cosmic ray experiment Alpha Magnetic Spectrometer AMS-02 on the International Space Station [18] (a prototype AMS-01 was flown on the Space Shuttle mission STS-91 in 1998). AMS-02 is a roughly 1 m^2 sterad detector which will analyze the flux of cosmic ray nuclei and particles in unprecedented detail for three years or more following deployment in 2005. It will be sensitive to strangelets in a wide range of mass, charge and energy [19].

Assuming that strangelets share two of the features found experimentally for cosmic ray nuclei, namely a power law energy distribution: $N(E)dE \propto E^{-2.5}$, and an average confinement time in the galaxy of 10^7 years, and including a geomagnetic cutoff rigidity of 6 GeV/c, the strangelet flux at AMS-02 would be

$$F \approx 5 \times 10^5 (\mathrm{m}^2 \text{ y sterad})^{-1} \times R_{-4} \times M_{-2} \times V_{100}^{-1} \times t_7, \tag{6}$$

where R_{-4} is the number of strange star collisions in our Galaxy per 10^4 years, M_{-2} is the mass of strangelets ejected per collision in units of $10^{-2} M_\odot$, V_{100} is the effective galactic volume in units of 100 kpc^3 over which strangelets are distributed, and t_7 is the average confinement time in units of 10^7 years. All these factors could be of order unity if strange matter is absolutely stable, though each with significant uncertainties. The flux estimate assumes a charge-mass relation $Z = 0.3A^{2/3}$ as derived for color-flavor locked strangelets, and is valid for $A < 6 \times 10^6$.

Strangelet propagation in the Milky Way Galaxy is in many ways expected to be similar to that of ordinary cosmic ray nuclei. Except for a possible background of slow-moving electrically neutral quark nuggets confined solely by the gravitational potential of the Galaxy, strangelets are charged and are therefore bound to the galactic magnetic field. They lose kinetic energy by electrostatic interactions with the interstellar medium, and they gain energy by Fermi acceleration in shock waves, for example from supernovae. Even if accelerated to relativistic speeds, scatterings on impurities in the magnetic field makes the motion resemble a diffusion process. The solar wind as well as the Earth's magnetic field become important for understanding the final approach to the detector. Also, strangelets may undergo spallation in collisions with cosmic ray nuclei, nuclei in the interstellar medium, or other strangelets. Much depends on the charge-to-mass relation, but the details of propagation are not even well understood for ordinary nuclei, so clearly some uncertainty in the expectations for the strangelet flux at AMS is inevitable.

A discovery of strangelets would be a very significant achievement; an ultimate smoking gun for the quark-gluon plasma at non-zero chemical potential with profound implications for the astrophysics of compact stars. Experimental information on the charge-to-mass relation may even allow a test of color-flavor locking in quark matter. Many uncertainties are clearly involved in the calculation of the strangelet flux at AMS-02. A systematic study of these issues has been initiated and should significantly improve our understanding of the strangelet production and propagation. But ultimately we must rely on experiment. It is reassuring, that the simple flux estimates above lead us to expect a very significant strangelet flux in the AMS-02 experiment.

Acknowledgments

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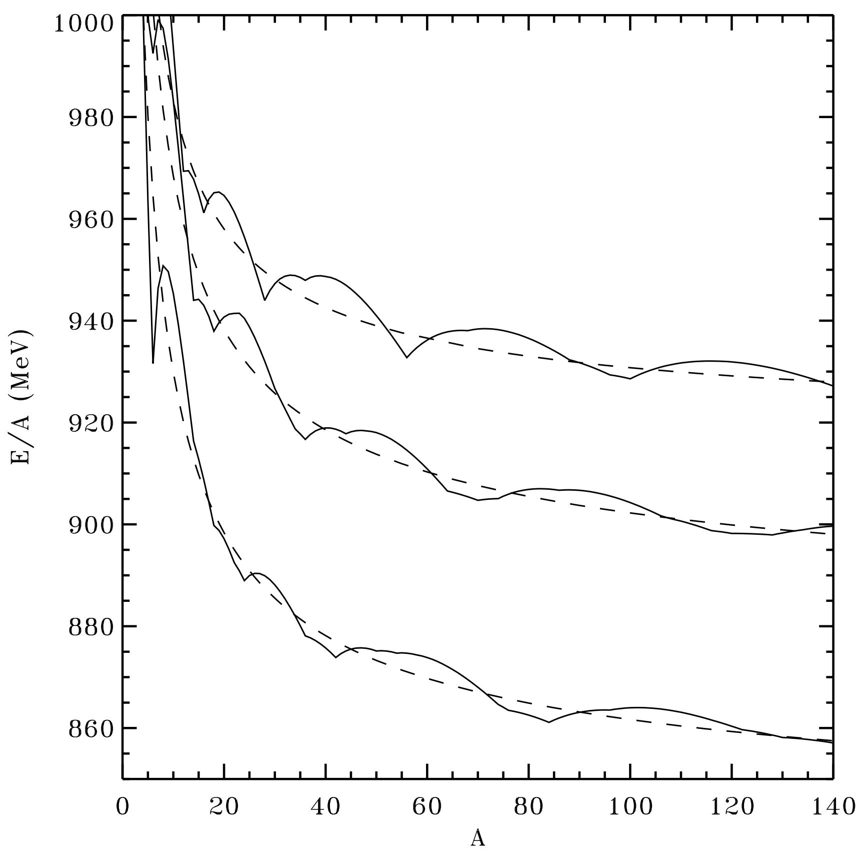

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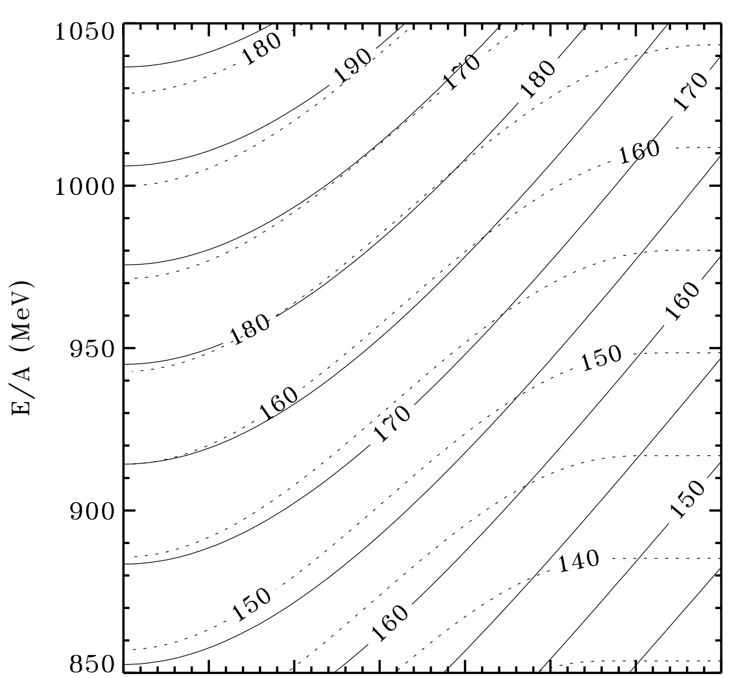

E/A (MeV)
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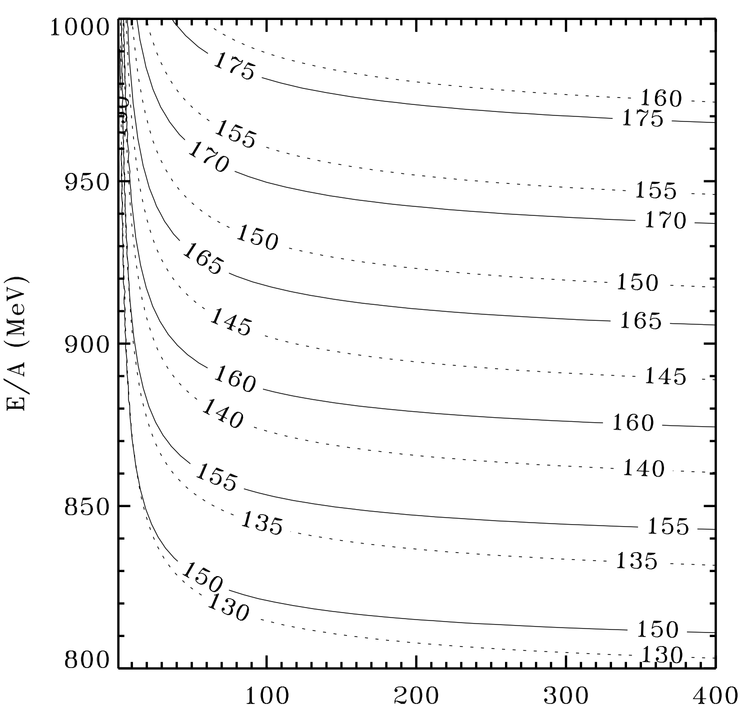
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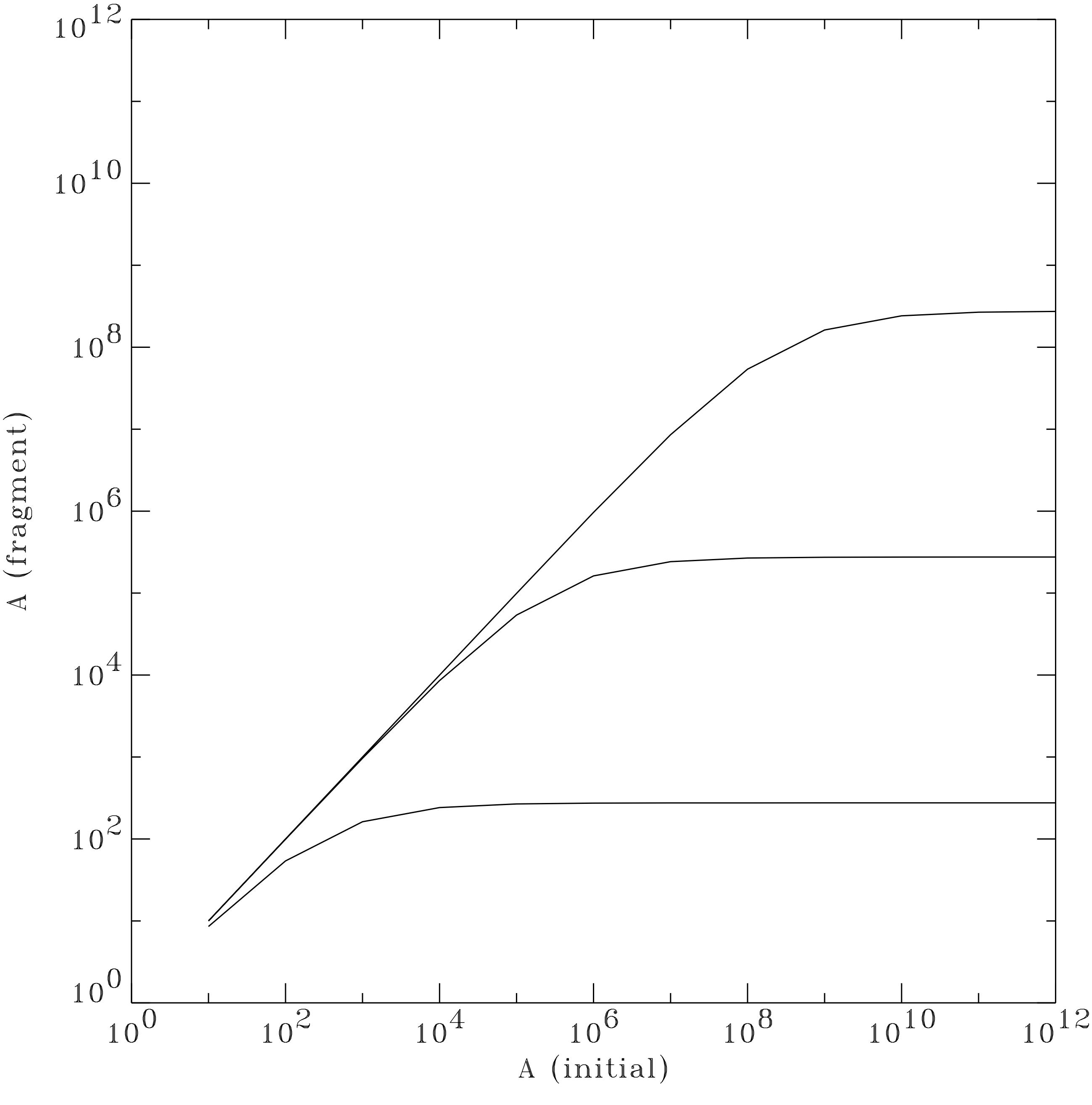

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A (fragment)
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A (initial)